6th Canadian Conf. Earthquake Engineering / Toronto / 1991 / 6ème Conf. Canadienne Sismique

Uncertainty in system modeling for seismic performance of structural systems

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ABSTRACT

This paper examines the effects of uncertainty in structural and material models on seismic response and reliability of structural systems. The analysis involves hysteretic constitutive laws commonly used in earthquake engineering to model restoring forces and extensive Monte Carlo simulation for obtaining probabilistic characteristics. Several numerical examples based on single- and multi-degree-of-freedom systems are presented.

INTRODUCTION

It is widely recognized that the seismic performance of structural systems is overwhelmingly dominated by the uncertainty in seismic load processes. Accordingly, reliability analysis is carried out by assuming deterministic structural and material characteristics thus ignoring their inherent stochasticity. The uncertainty in system modeling is usually present due to the variabilities in (*i*) mathematical idealization of structural system, (*ii*) mathematical representation of hysteretic restoring forces, and (*iii*) the parameters of restoring force characteristics given a hysteretic model [3,4].

This paper conducts a systematic investigation to determine the effects of uncertainty in system modeling on seismic response and reliability of structural systems. The method of analysis is based on (*i*) common hysteretic constitutive laws for material models and (*ii*) extensive Monte Carlo simulation for performance evaluation. Several numerical examples on single- and multi-degree-of-freedom systems are presented.

STRUCTURAL SYSTEMS

In predicting the response and damage of actual structures, modeling of structural systems is an essential task. A model of a structure is defined as a mathematical representation of the behavior of the structure in its environment. The accuracy of response prediction depends on how well the models approximate the actual behavior of the structure.

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While derivation of the governing field equations of continuous models is not unduly difficult, the attainment of general solution is a formidable task. To date, analytic solutions are known only for a few relatively simple continuous systems with linear elastic constitutive law, e.g., uniform beams, strings, plates and shells with simple boundary conditions. For the dynamic analysis of skeletal structures like frames, the continuous models becomes extremely complex and have thus found limited use in practice.

Consider a discrete, nonlinear structural system with critical cross-sections associated with appropriate restoring forces. The seismic modeling of this multi-degree-of-freedom, hysteretic system leads to the matrix differential equations of the form [3]

$$\mathbf{m}\ddot{\mathbf{X}}(t) + \mathbf{g}\left(\{\mathbf{X}(s), \dot{\mathbf{X}}(s), 0 \leq s \leq t\}; t\right) = -\mathbf{m}\mathbf{d}W(t) \tag{1}$$

with the initial conditions

$$\mathbf{X}(0) = \mathbf{0}, \quad \text{and} \quad \dot{\mathbf{X}}(0) = \mathbf{0} \tag{2}$$

where t is the time coordinate originating at the beginning of seismic event $W(t)$, $\mathbf{X}(t)$ is a vector of generalized displacements, $\mathbf{g}$ is the vector functional representing general nonlinear hysteretic restoring forces, $\mathbf{m}$ is the constant mass matrix, and $\mathbf{d}$ is a vector of influence coefficients. In earthquake engineering, the total restoring force $\mathbf{g}$ is usually allowed to admit an additive decomposition of a nonhysteretic component

$$\mathbf{g}_{nh} = \mathbf{c}\,\dot{\mathbf{X}}(t) + \mathbf{k}_{nh}\left(\mathbf{X}(t)\right)\mathbf{X}(t) \tag{3}$$

and a hysteretic component

$$\mathbf{g}_{h} = \mathbf{k}_{h}\left(\mathbf{Z}(t)\right)\mathbf{X}(t) \tag{4}$$

where $\mathbf{c}$ is the constant viscous damping matrix, $\mathbf{k}_{nh}$ is the nonhysteretic part of stiffness matrix, $\mathbf{k}_h$ is the hysteretic part of stiffness matrix, and $\mathbf{Z}(t)$ is the vector of additinal hysteretic variables the time evolution of which can be modeled by a set of general nonlinear ordinary differential equations

$$\dot{\mathbf{Z}}(t) = \mathbf{F}\left(\mathbf{X}(t), \dot{\mathbf{X}}(t), \mathbf{Z}(t); t\right) \tag{5}$$

in which $\mathbf{F}$ is a general nonlinear vector function the explicit expression of which depends on the hysteretic rule governed by a particular constitutive law. Following the state vector approach with the designation of $\theta_1(t) = \mathbf{X}(t)$, $\theta_2(t) = \dot{\mathbf{X}}(t)$, and $\theta_3(t) = \mathbf{Z}(t)$ the equivalent system of first-order nonlinear differential equations in state variables become

$$\begin{aligned}
\dot{\theta}_1(t) &= \theta_2(t) \\
\dot{\theta}_2(t) &= -\mathbf{m}^{-1}\left[\mathbf{c}\,\theta_2(t) + \mathbf{k}_{nh}\left(\theta_1(t)\right)\theta_1(t) + \mathbf{k}_h\left(\theta_3(t)\right)\theta_1(t)\right] - \mathbf{d}\,W(t) \\
\dot{\theta}_3(t) &= \mathbf{F}\left(\theta_1(t), \theta_2(t), \theta_3(t); t\right)
\end{aligned} \tag{6}$$

which can be recast in a more compact form

$$\dot{\theta}(t) = \mathbf{h}\left(\theta(t); t\right) \tag{7}$$

with the initial conditions $\theta(0) = 0$ where $h(\cdot)$ is a vector function and $\theta(t) = \{\theta_1(t), \theta_2(t), \theta_3(t)\}^T$ is the response state vector. The nonlinear system of first-order ordinary differential equations in the initial-value problem of Eq. 7 can be solved by using step-by-step numerical integration such as fifth- and sixth-order Runge-Kutta integrators. When the excitation and/or the structural and material characteristics are random, $\theta(t)$ becomes a vector stochastic process which characterizes state of structural system.

UNCERTAINTY IN STRUCTURAL MODELS

A major source of seismic risk in New York City relates to the hundreds of flat slab apartment buildings constructed during the past 40 years. These buildings house thousands of people and were designed primarily for gravity loads. Fig. 1 shows a floor plan of a 24-story R/C flat-slab building.

The structure is modeled as a two-dimensional frame-shear wall type building based on the assumption that the floors have perfect in-plane rigidity. Moment of inertia of all the columns are lumped into columns of a 3-bay planar frame (System-A). For the shear walls, the moment of inertia are lumped into two separate walls (System-B and -C) corresponding to contributions from small and large walls. Hinged links are then used to transfer the axial loads from System-A to System-B and then from System-B to System-C. The simplified idealized structure is shown in Fig. 2.

Out-of-plane bending of floor slabs are considered by idealizing slabs into equivalent beams [5] of same depth with effective width being some fraction λ_w (effective width coefficient) of slab panel width. Using the chart used in Ref. 5 with proper regard to the irregularity of plan, a lower bound of $\lambda_w = 0.35$ and an upper bound of $\lambda_w = 1.0$ are obtained.

The variability of effective width coefficient λ_w may incorporate substantial amount of uncertainty in the response of structure due to earthquake loads. For example, the initial fundamental natural period T_0 of the building is 2.9 s for $\lambda_w = 0.35$ and 2.3 s for $\lambda_w = 1.0$. Fig. 3 shows a plot of top displacement of the building versus seismic base shear coefficient obtained from nonlinear static analysis based on a bilinear force-deformation model [5]. Significant differences are noticed in the values of maximum base shear coefficients, *e.g.* 0.045 and 0.08 when calculated for $\lambda_w = 0.35$ and $\lambda_w = 1.0$, respectively.

UNCERTAINTY IN MATERIAL MODELS

The variability in material models constitutes another major source of uncertainty in the evaluation of seismic performance of structural systems. Two sources can be identified and they correspond to the uncertainty in (*i*) the mathematical idealization of hysteretic constitutive law and (*ii*) the parameters of restoring force characteristics given a hysteretic model.

Single-Degree-of-Freedom Systems

Consider a nonlinear hysteretic oscillator with mass $m = 1.0$ $kips$ s^2 in^{-1}, damping coefficient $c = 0.06$ $kips$ s in^{-1}, initial stiffness $k = 1.0$ $kips$ in^{-1}, yield strength $F_y = 1.0$ $kips$ which is subjected to a *zero*-mean stationary Gaussian random process $W(t)$ with one-sided power spectral density $G(\omega) = G_0$ for $\omega \leq \bar{\omega} = 3$ rad s^{-1} and *zero* otherwise. The duration of motion is assumed to be $t_d = 40$ s.

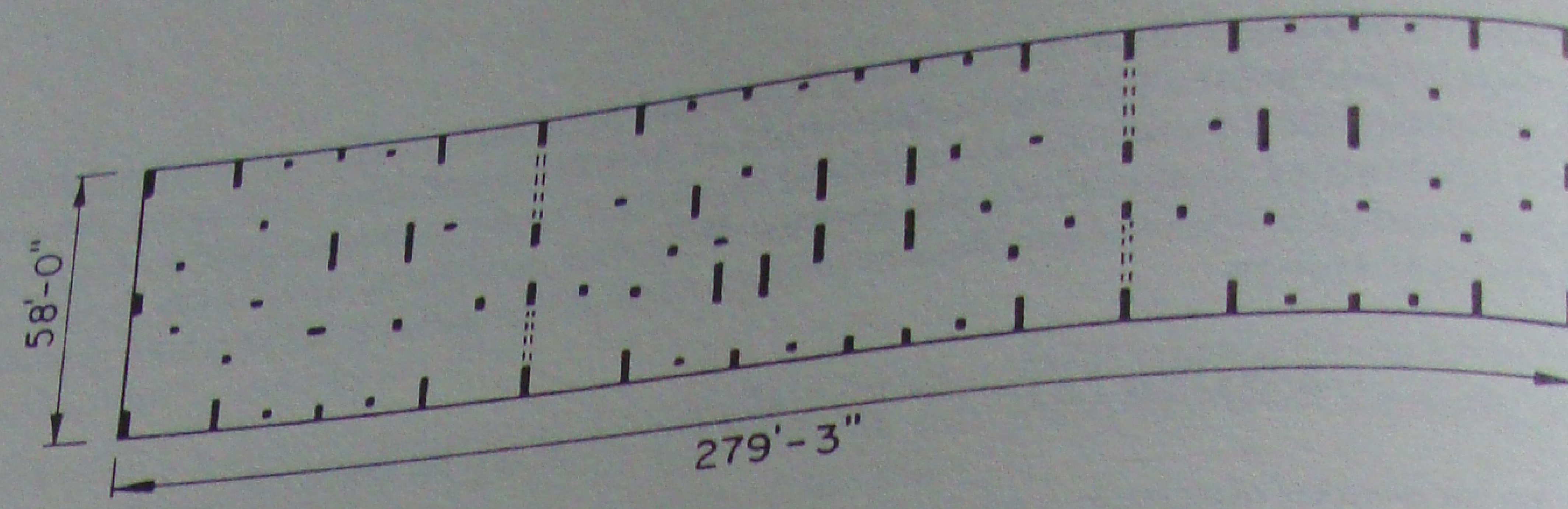

Fig. 1: Typical Floor Plan of Building System

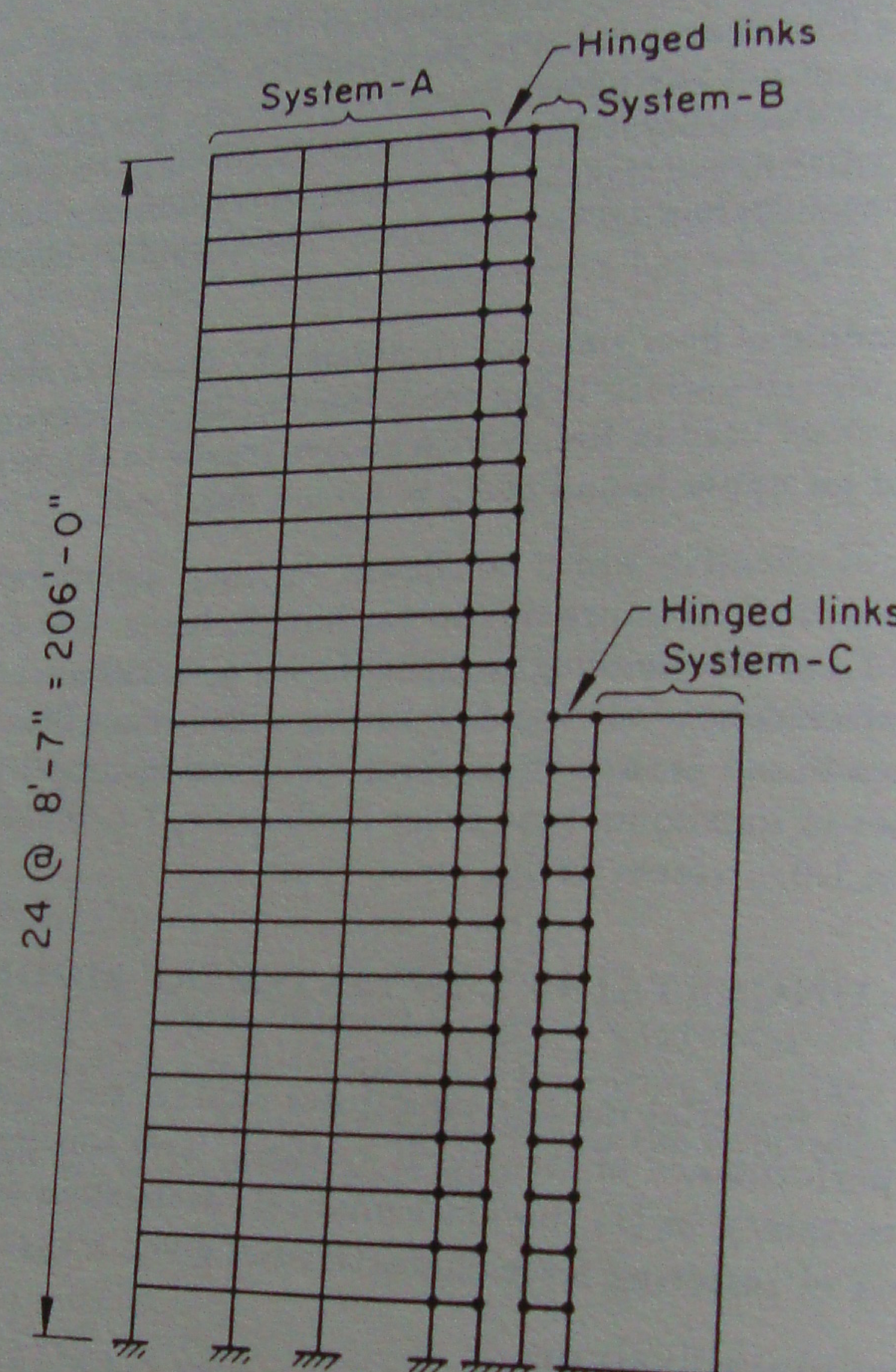

Fig. 2: Idealized Frame-Wall System

Three nondegrading restoring force models are considered in the sensitivity study: the ideal elasto-plastic, bilinear and Bouc-Wen. Details of these hysteretic models are available in the current literature [1,3,7]. Explicit description of the associated F functions in Eq. 5 can be obtained from Ref. 3. The models are equivalent in the sense that they have the same initial stiffness and strength characteristics and the parameters of each model are assumed to be deterministic.

Seismic performance of structural systems can be evaluated in terms of the condition that a specific response or damage level is exceeded during seismic ground motion. One such response quantity of interest is the ductility ratio μ given by

$$\mu = \frac{1}{X_y}\left[\max_t X(t)\right] \tag{8}$$

where X_y is the yield displacement and $X(t)$ is the relative displacement response of the oscillator. Since the ground motion is modeled as random process, a realistic assessment requires computation of the the probability $p = \Pr(\mu > \mu_0)$ with μ_0 representing the ductility threshold. This probability is estimated in the paper by Monte Carlo simulation with 3000 samples.

Fig. 4 shows the probability $p = \Pr(\mu > \mu_0)$ for several intensities of the input. For weak noise ($G_0 = 0.005\ in^2\ s^{-3}$), the exceedance probability of μ for Bouc-Wen hysteresis is considerably smaller than that for either elasto-plastic or bilinear models which exhibit identical behavior due to mostly linear response. For strong noise ($G_0 = 0.5\ in^2\ s^{-3}$), the probabilities $\Pr(\mu > \mu_0)$ becomes similar for bilinear and Bouc-Wen models both of which show smaller values of above probability than that for the elasto-plastic model. When the strength of noise is somewhat intermediate ($G_0 = 0.5\ in^2\ s^{-3}$), all the hysteretic models exhibit practically similar behavior.

Multi-Degree-of-Freedom Systems

Consider a 10-story steel frame building in Ref. 2 which is idealized here as a 10-degree-of-freedom shear beam system (stick model) with one degree of freedom per story. The mean values for the physical properties of stick model obtained from Refs. 2 and 4. Mass, damping coefficient, stiffness, and strength are treated as independent lognormal random variables with coefficients of variation 11%, 65%, 13%, and 23%, respectively. The coefficients of variation account for uncertainty in both mathematical idealization of hysteretic models and model parameters and are obtained from Ref. 6. Due to common construction and workmanship, each of these random variables are assumed to be perfectly correlated among all the stories. The restoring force at each story k is modeled by the nondegrading Bouc-Wen hysteresis with the hysteretic parameters defined in Ref. 4.

The ground motion is modeled as a uniformly modulated random process $W(t) = \psi(t)\widetilde{W}(t)$ where the modulation function

$$\psi(t) = \begin{cases} \left(\frac{t}{t_1}\right)^2, & 0 \le t \le t_1 \\ 1, & t_1 \le t \le t_2 \\ \exp[-c_\psi(t - t_2)], & t_2 \le t \end{cases} \tag{9}$$

and the $\widetilde{W}(t)$ is a *zero*-mean stationary Gaussian colored process with one-sided power spectral density

$$\widetilde{G}(\omega) = G_0 \frac{1 + \left[2\zeta_g(\frac{\omega}{\omega_g})\right]^2}{\left[1 - (\frac{\omega}{\omega_g})^2\right]^2 + \left[2\zeta_g(\frac{\omega}{\omega_g})\right]^2} \tag{10}$$

in which $t_1 = 1.5\ s$, $t_2 = 8.5\ s$, $c_\psi = 0.18\ s^{-1}$, $\omega_g = 16.5\ rad\ s^{-1}$, $\zeta_g = 0.8$, and $G_0 = 130.0\ in^2\ s^{-3}$. The duration of motion is assumed to be $t_d = 15\ s$.

Table 1 shows the exceedance probability of story level ductility ratio μ_k $(k = 1, 2, \cdots, 10)$ for several thresholds $\mu_0 = 3, 4, 5, 6$ at different stories of the 10-story steel frame structure. Two cases are considered. In the first case, the analysis is based on deterministic structural and material characteristics obtained from the mean values in Ref. 2. In the second case, the analysis accounts for the uncertainty in structural system with its probabilistic characteristics mentioned earlier. In both cases, Monte Carlo simulation is performed with 3000 samples. Results from Table 1 suggest that the uncertainty in system modeling can increase significantly the exceedance probability of story ductility.

CONCLUSIONS

A systematic investigation is conducted to study the sensitivity of seismic performance to the uncertainty in structural and material characteristics. The analysis is based on commonly used hysteretic constitutive laws for modeling restoring forces and extensive Monte Carlo simulation for seismic performance evaluation.

Results from a 24-story R/C flat-slab building suggest that the variability in structural model itself can significantly alter dynamic characteristics of structural system. Reliability analysis performed on a single-degree-of-freedom system and a 10-story steel frame building reveal that the material uncertainty can also have significant effect on seismic response and reliability.

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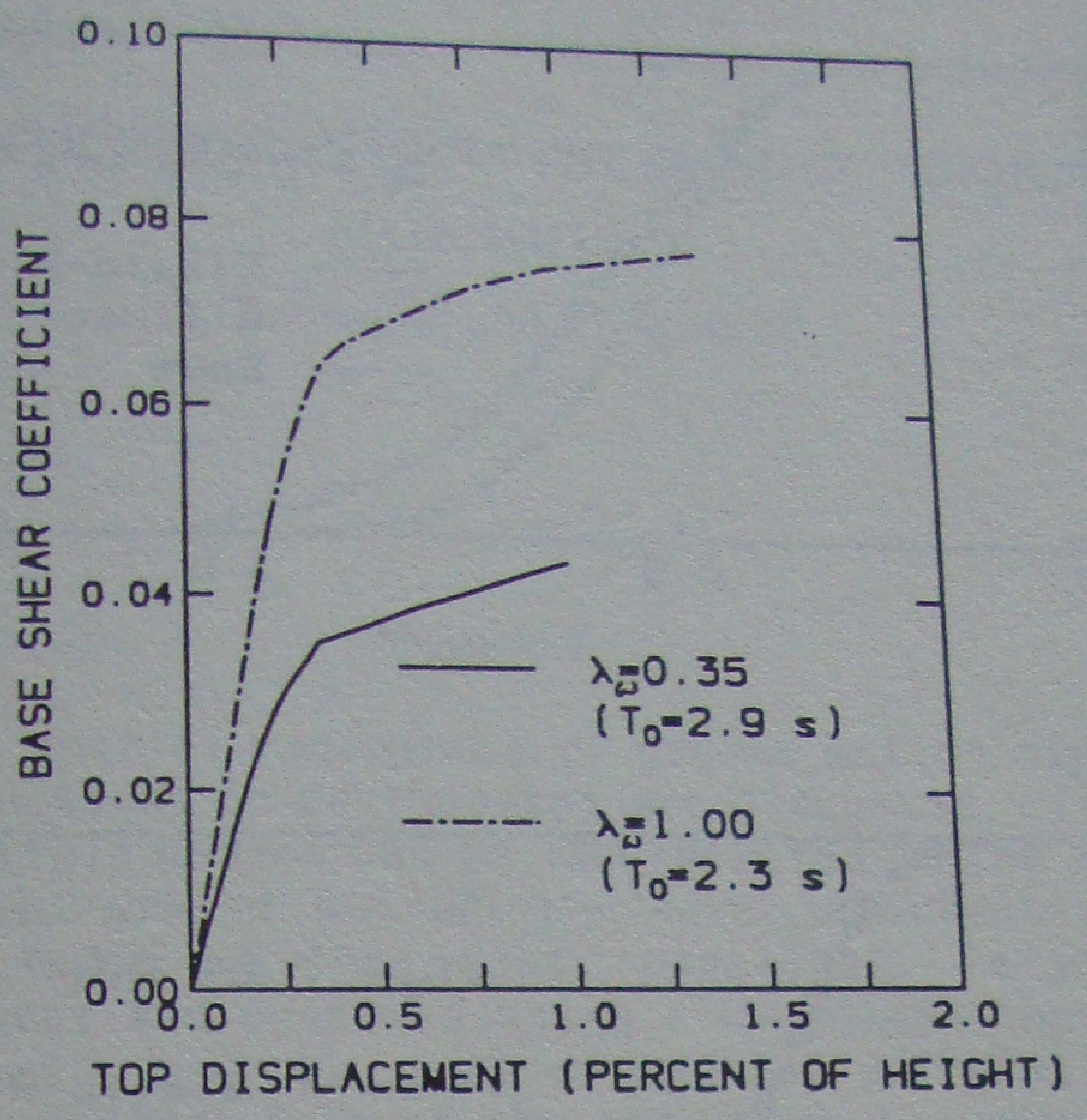

Fig. 3: Base Shear Coefficient Versus Top Displacement

Table 1: Exceedance Probability of Ductility Ratio

Cases	story k	$Pr(\mu_k > \mu_0)$			
		$\mu_0 = 3$	$\mu_0 = 4$	$\mu_0 = 5$	$\mu_0 = 6$
Deterministic System	1	0.383000	0.151000	5.666×10^{-2}	2.233×10^{-2}
	2	0.201333	6.266×10^{-2}	1.566×10^{-2}	5.333×10^{-3}
	3	0.121000	2.400×10^{-2}	6.000×10^{-3}	1.666×10^{-3}
	4	7.633×10^{-2}	1.166×10^{-2}	2.000×10^{-3}	3.333×10^{-4}
	5	4.500×10^{-2}	5.666×10^{-3}	0.000	0.000
	6	4.400×10^{-2}	3.000×10^{-3}	0.000	0.000
	7	3.133×10^{-2}	9.999×10^{-4}	0.000	0.000
	8	0.000	0.000	0.000	0.000
	9	0.000	0.000	0.000	0.000
	10	0.000	0.000	0.000	0.000
Uncertain System	1	0.452666	0.269000	0.154333	9.099×10^{-2}
	2	0.312666	0.153000	7.733×10^{-2}	4.133×10^{-2}
	3	0.223000	9.200×10^{-2}	4.066×10^{-2}	2.200×10^{-2}
	4	0.161000	5.700×10^{-2}	2.433×10^{-2}	1.033×10^{-2}
	5	0.109333	3.466×10^{-2}	1.066×10^{-2}	5.000×10^{-3}
	6	0.114000	3.066×10^{-2}	7.333×10^{-3}	3.666×10^{-3}
	7	0.104666	2.133×10^{-2}	5.000×10^{-3}	1.333×10^{-3}
	8	1.333×10^{-3}	0.000	0.000	0.000
	9	9.999×10^{-4}	0.000	0.000	0.000
	10	0.000	0.000	0.000	0.000

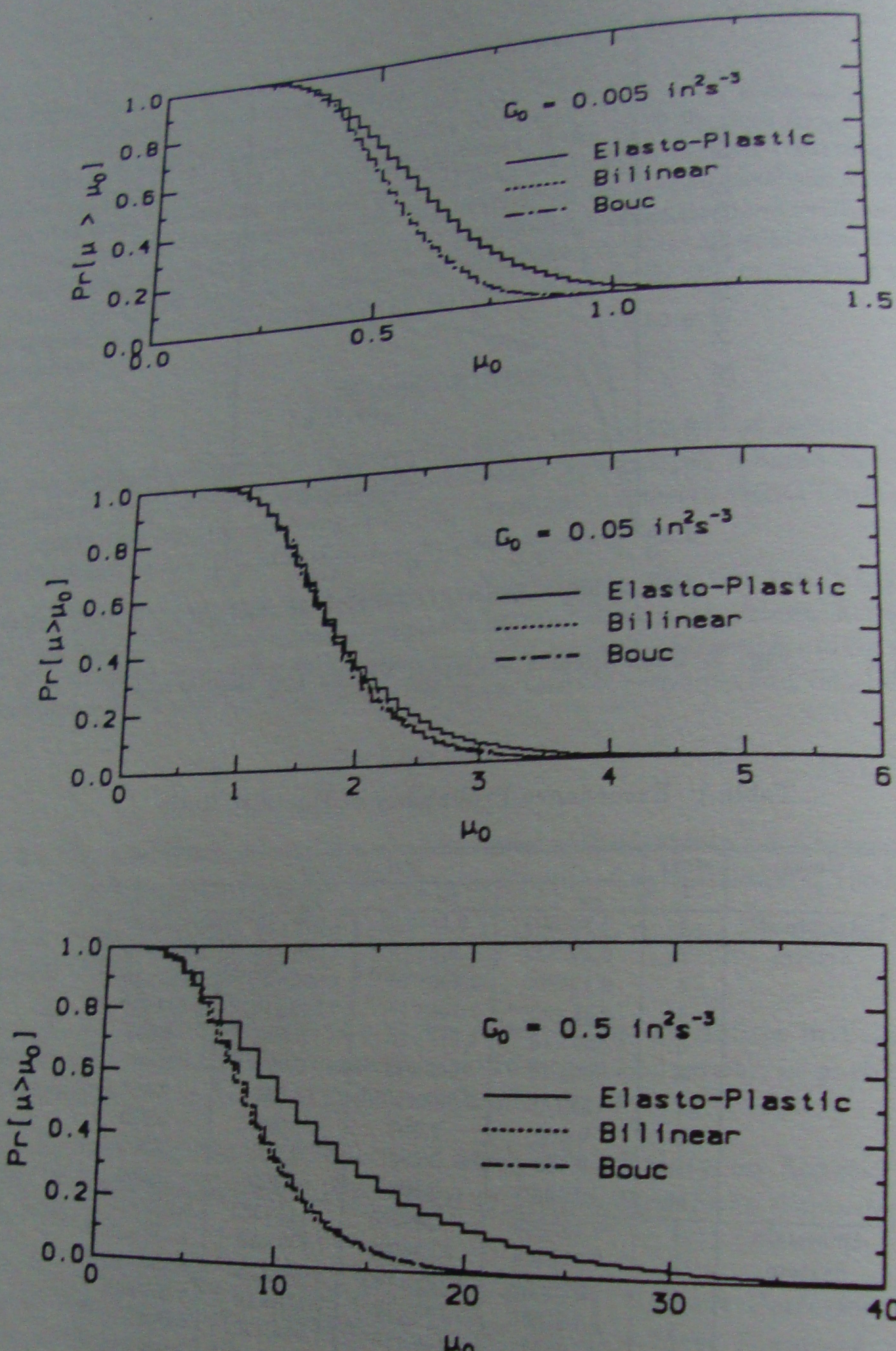

Fig. 4: Exceedance Probability of Ductility Ratio